1 Steady state linear elasticity

The steady state linear elasticity equations will be considered on the domain Ω. The boundary Γ consists of a section associated with the Dirichlet boundary condition denoted by Γ_D and a section associated with the Neumann boundary condition denoted by Γ_N. The boundary $\Gamma = \Gamma_N \cup \Gamma_D$.

1.1 Strong form

The strong form of the governing equations along with boundary conditions states the conditions at every point over a domain that a solution must satisfy.

$$
\begin{aligned}
\text{div}\boldsymbol{\sigma} + \mathbf{f} &= \mathbf{0}, \quad \text{on} \quad \Omega && (1)\\
\boldsymbol{\sigma} - \mathbf{C}\boldsymbol{\varepsilon} &= 0, && (2)\\
\mathbf{t} - \boldsymbol{\sigma} \cdot \mathbf{n} &= 0, \quad \text{on} \quad \Gamma_N && (3)\\
\mathbf{u} &= \bar{\mathbf{u}}, \quad \text{on} \quad \Gamma_D && (4)
\end{aligned}
$$

equations (1), (2), (3) and (4) refer to the equilibrium equation, stress strain relation, traction boundary condition, and displacement boundary condition respectively.

1.2 Weak form

w is the test function. Equation (5) illustrates the weak form.

$$
\int_{\Omega} \boldsymbol{\nabla}\mathbf{w} \cdot \boldsymbol{\sigma} \; d\Omega = \int_{\Gamma_N} \mathbf{w} \cdot \boldsymbol{t} \; d\Gamma + \int_{\Omega} \mathbf{w} \cdot \boldsymbol{f} \; d\Omega \qquad (5)
$$