q
p
,
q
]
ψ
=
$\hbar$
i
ψ
∂
q
μ
ϵ
μ
)

If you make some rather abstract assumptions about the physics, then you can derive the HUP. You assume that the state of a physical system is described by a normalized vector ψ in a complex valued vector space of high dimension. This vector is a function of the physical variables, q, of the system, $\psi(q_\mu)$. So the values of physical variables are given by projections of this state vector onto the axes of the vector space. Then we consider unitary transformations, $\psi' = U\psi$ of the state vector. Unitary means the norm isn't changed, so it's a rotation, $U = \exp(i\theta)$. In two dimensions θ is just an angle, but in this higher dimensioned space it needs to be an operator, i.e. a matrix. Let it be an infinitesimal transformation $\theta = -\epsilon G$. Then the infinitesimal unitary transformation is given by $U = 1 - i\epsilon G$ where G is called the generator of the transformation.

$\psi' = \psi - i\epsilon G\psi$

Then we compare this to the form of a change of basis, $q'_\mu = q_\mu - \epsilon_\mu$

$\psi'(q_\mu') = \psi(q_\mu - \epsilon_\mu)$

$\quad = \psi(q_\mu) - \epsilon_\mu \partial\psi(q_\mu)/\partial q_\mu$

It is seen that the generator is

$G_\mu = -i\partial/\partial q_\mu$

Then to make this look more familiar we define

$P_\mu \equiv \hbar G_\mu = -i\hbar\partial/\partial q_\mu$

Note that $\hbar$ is just an arbitrary constant we introduce because we want to measure P in units other than the inverse of q. Then it follows that

$[p, q]\psi = -i\hbar\psi$

So Heisenberg's uncertainy relation comes out of the mathematics AFTER you've put some assumptions about how physical systems are represented in a high dimensional vector space by a normalized vector that just rotates. The HUP is just the statement that the infinitesimal rotation corresponding to a change in a variable q fails to commute with the conjugate variable p, we call "momentum" along q, which is not surprising since rotations don't commute in general.

1 view

1 of 6 answers

Upvote