Q32
Does the EPR experiment prohibit locality? What about Bell's Inequality?

The EPR experiment is widely regarded as the definitive gedanken experiment for demonstrating that quantum mechanics is non-local (requires faster-than-light communication) or incomplete. We shall see that it implies neither.

The EPR experiment was devised, in 1935, by Einstein, Podolsky and Rosen to demonstrate that quantum mechanics was incomplete [E]. Bell, in 1964, demonstrated that any hidden variables theory, to replicate the predictions of QM, must be non-local [B]. QM predicts strong correlations between separated systems, stronger than any local hidden variables theory can offer. Bell encoded this statistical prediction in the form of some famous inequalities that apply to any type of EPR experiment. Eberhard, in the late 1970s, extended Bell's inequalities to cover any local theory, with or without hidden variables. Thus the EPR experiment plays a central role in sorting and testing variants of QM. All the experiments attempting to test EPR/Bell's inequality to date (including Aspect's in the 1980s [As]) are in line with the predictions of standard QM - hidden variables are ruled out. Here is the paradox of the EPR experiment. It seems to imply that any physical theory must involve faster-than-light "things" going on to maintain these "spooky" action-at-a-distance correlations and yet still be compatible with relativity, which seems to forbid FTL.

Let's examine the EPR experiment in more detail.

So what did EPR propose? The original proposal was formulated in terms of correlations between the positions and momenta of two once-coupled particles. Here I shall describe it in terms of the spin (a type of angular momentum intrinsic to the particle) of two electrons. [In this treatment I shall ignore the fact that electrons always form antisymmetric combinations. This does not alter the results but does simplify the maths.] Two initially coupled electrons, with opposed spins that sum to zero, move apart from each other across a distance of perhaps many light years, before being separately detected, say, by me on Earth and you on Alpha Centauri with our respective measuring apparatuses. The EPR paradox results from noting that if we choose the same (parallel) spin axes to measure along then we will observe the two electrons' spins to be anti-parallel (i.e. when we communicate we find that the spin on our electrons are correlated and opposed). However if we choose measurement spin axes that are perpendicular to each other then there is no correlation between electron spins. Last minute alterations in a detector's alignment can create or destroy correlations across great distances. This implies, according to some theorists, that faster-than-light influences maintain correlations between separated systems in some circumstances and not others.

Now let's see how many-worlds escapes from this dilemma.

The initial state of the wavefunction of you, me and the electrons and the rest of the universe may be written:

```
   |psi> =  |me> |electrons> |you> |rest of universe>
             on      in       on
            Earth   deep     Alpha
                    space   Centauri
or more compactly, ignoring the rest of the universe, as:
```

```
   |psi> =  |me, electrons, you>
And
     |me> represents me on Earth with my detection apparatus.
     |electrons> = (|+,-> - |-,+>)/sqrt(2)
        represents a pair electrons, with the first electron travelling
        towards Earth and the second electron travelling towards Alpha
        Centauri.

   |+> represents an electron with spin in the +z direction
   |-> represents an electron with spin in the -z direction
```

It is an empirically established fact, which we just have to accept, that we can relate spin states in one direction to spin states in other directions like so (where "i" is the sqrt(-1)):

```
   |left>  = (|+> - |->)/sqrt(2)    (electron with spin in -x direction)
   |right> = (|+> + |->)/sqrt(2)    (electron with spin in +x direction)
   |up>    = (|+> + |->i)/sqrt(2)   (electron with spin in +y direction)
   |down>  = (|+> - |->i)/sqrt(2)   (electron with spin in -y direction)
and inverting:
   |+>  = (|right> + |left>)/sqrt(2) =  (|up> + |down>)/sqrt(2)
   |->  = (|right> - |left>)/sqrt(2) =  (|down> - |up>)i/sqrt(2)
```

(In fancy jargon we say that the spin operators in different directions form non-commuting observables. I shall eschew such obfuscations.)

Working through the algebra we find that for pairs of electrons:

```
   |+,-> - |-,+> =  |left,right> -  |right,left>
                 =  |up,down>i    - |down,up>
```

I shall assume that we are capable of either measuring spin in the x or y direction, which are both perpendicular the line of flight of the electrons. After having measured the state of the electron my state is described as one of either:

```
   |me[l]> represents me + apparatus + records having measured
           and recorded the x-axis spin as "left"
   |me[r]> ditto with the x-axis spin as "right"
   |me[u]> ditto with the y-axis spin as "up"
   |me[d]> ditto with the y-axis spin as "down"
```

Similarly for |you> on Alpha Centauri. Notice that it is irrelevant *how* we have measured the electron's spin. The details of the measurement process are irrelevant. (See "What is a measurement?" if you're not convinced.) To model the process it is sufficient to assume that there is a way, which we have further assumed does not disturb the electron. (The latter assumption may be relaxed without altering the results.)

To establish familiarity with the notation let's take the state of the initial wavefunction as:

```
             |psi>_1 =  |me,left,up,you>
                             /     \
                           /         \
    first electron in left          second electron in up state
    state heading towards              heading towards you on
        me on Earth                         Alpha Centauri
```

After the electrons arrive at their detectors, I measure the spin along the x-axis and you along the y-axis. The wavefunction evolves into |psi>_2:

```
                local
      |psi>_1 ============> |psi>_2 = |me[l],left,up,you[u]>
                observation
```

which represents me having recorded my electron on Earth with spin left and you having recorded your electron on Alpha Centauri with spin up. The index in []s indicates the value of the record. This may be held in the observer's memory, notebooks or elsewhere in the local environment (not necessarily in a readable form). If we communicate our readings to each other the wavefunctions evolves into |psi>_3:

```
               remote
      |psi>_2 ============> |psi>_3 = |me[l,u],left,up,you[u,l]>
              communication
```

where the second index in []s represents the remote reading communicated to the other observer and being recorded locally. Notice that the results both agree with each other, in the sense that my record of your result agrees with your record of your result. And vice versa. Our records are consistent.

That's the notation established. Now let's see what happens in the more general case where, again,:

```
     |electrons> = (|+,-> - |-,+>)/sqrt(2).
```

First we'll consider the case where you and I have previously arranged to measure the our respective electron spins along the same x-axis.

Initially the wavefunction of the system of electrons and two experimenters is:

```
   |psi>_1
     =  |me,electrons,you>
     =  |me>(|left,right> - |right,left>)|you> /sqrt(2)
     =  |me,left,right,you> /sqrt(2)
      - |me,right,left,you> /sqrt(2)
```

Neither you or I are yet unambiguously split.

Suppose I perform my measurement first (in some time frame). We get

```
   |psi>_2
     =  (|me[l],left,right> - |me[r],right,left>)|you> /sqrt(2)
     =    |me[l],left,right,you> /sqrt(2)
        - |me[r],right,left,you> /sqrt(2)
```

My measurement has split me, although you, having made no measurement, remain unsplit. In the full expansion the terms that correspond to you are identical.

After the we each have performed our measurements we get:

```
   |psi>_3
     =  |me[l],left,right,you[r]> /sqrt(2)
      - |me[r],right,left,you[l]> /sqrt(2)
```

The observers (you and me) have been split (on Earth and Alpha Centauri) into relative states (or local worlds) which correlate with the state of the electron. If we now communicate over interstellar modem (this will take a few years since you and I are separated by light years, but no matter). We get:

```
|psi>_4
  =  |me[l,r],left,right,you[r,l]> /sqrt(2)
   - |me[r,l],right,left,you[l,r]> /sqrt(2)
```

The world corresponding to the 2nd term in the above expansion, for example, contains me having seen my electron with spin right and knowing that you have seen your electron with spin left. So we jointly agree, in both worlds, that spin has been conserved.

Now suppose that we had prearranged to measure the spins along different axes. Suppose I measure the x-direction spin and you the y-direction spin. Things get a bit more complex. To analyse what happens we need to decompose the two electrons along their respective spin axes.

```
|psi>_1 =
     |me,electrons,you>
  = |me>(|+,-> - |-,+>)|you>/sqrt(2)
  = |me> (
          (|right>+|left>)i(|down>-|up>)
        - (|right>-|left>)(|down>+|up>)
         ) |you> /2*sqrt(2)
  = |me> (
          |right>(|down>-|up>)i
        + |left> (|down>-|up>)i
        - |right>(|down>+|up>)
        + |left> (|down>+|up>)
         ) |you> /2*sqrt(2)
  = |me> (
          |right,down> (i-1) - |right,up> (1+i)
        + |left,up> (1-i)    + |left,down> (1+i)
         ) |you> /2*sqrt(2)
  =  (
     + |me,right,down,you> (i-1)
     - |me,right,up,you>   (i+1)
     + |me,left,up,you>    (1-i)
     + |me,left,down,you>  (1+i)
     ) /2*sqrt(2)
```

So after you and I make our local observations we get:

```
|psi>_2 =
     (
     + |me[r],right,down,you[d]> (i-1)
     - |me[r],right,up,you[u]>   (i+1)
     + |me[l],left,up,you[u]>    (1-i)
     + |me[l],left,down,you[d]>  (1+i)
     ) /2*sqrt(2)
```

Each term realises a possible outcome of the joint measurements. The interesting thing is that whilst we can decompose it into four terms there are only two states for each observer. Looking at myself, for instance, we can rewrite this in terms of states relative to *my* records/memories.

```
|psi>_2 =
     (
       |me[r],right> ( |down,you[d]> (i-1) - |up,you[u]> (i+1) )
     + |me[l],left>  ( |up,you[u]> (1-i) + |down,you[d]> (1+i) )
```

```
    ) /2*sqrt(2)
```

And we see that there are only two copies of *me*. Equally we can rewrite the expression in terms of states relative to *your* records/memory.

```
|psi>_2 =
    (
      ( |me[l],left> (1-i) - |me[r],right> (i+1) ) |up,you[u]>
    + ( |me[r],right> (i-1) + |me[l],left> (1+i) ) |down,you[d]>
    ) /2*sqrt(2)
```

And see that there are only two copies of *you*. We have each been split into two copies, each perceiving a different outcome for our electron's spin, but we have not been split by the measurement of the remote electron's spin.

After you and I communicate our readings to each other, more than four years later, we get:

```
|psi>_3 =
    (
    + |me[r,d],right,down,you[d,r]> (i-1)
    - |me[r,u],right,up,you[u,r]>   (i+1)
    + |me[l,u],left,up,you[u,l]>    (1-i)
    + |me[l,d],left,down,you[d,l]>  (1+i)
    ) /2*sqrt(2)
```

The decomposition into four worlds is forced and unambiguous after communication with the remote system. Until the two observers communicated their results to each other they were each unsplit by each others' measurements, although their own local measurements had split themselves. The splitting is a local process that is causally transmitted from system to system at light or sub-light speeds. (This is a point that Everett stressed about Einstein's remark about the observations of a mouse, in the Copenhagen interpretation, collapsing the wavefunction of the universe. Everett observed that it is the mouse that's split by its observation of the rest of the universe. The rest of the universe is unaffected and unsplit.)

When all communication is complete the worlds have finally decomposed or decohered from each other. Each world contains a consistent set of observers, records and electrons, in perfect agreement with the predictions of standard QM. Further observations of the electrons will agree with the earlier ones and so each observer, in each world, can henceforth regard the electron's wavefunction as having collapsed to match the historically recorded, locally observed values. This justifies our operational adoption of the collapse of the wavefunction upon measurement, without having to strain our credibility by believing that it actually happens.

To recap. Many-worlds is local and deterministic. Local measurements split local systems (including observers) in a subjectively random fashion; distant systems are only split when the causally transmitted effects of the local interactions reach them. We have not assumed any non-local FTL effects, yet we have reproduced the standard predictions of QM.

So where did Bell and Eberhard go wrong? They thought that all theories that reproduced the standard predictions must be non-local. It has been pointed out by both Albert [A] and Cramer [C] (who both support different interpretations of QM) that Bell and Eberhard had implicity assumed that every possible measurement - even if not performed - would have yielded a *single* definite result. This assumption is called contra-factual definiteness or CFD [S]. What Bell and Eberhard really proved was that every quantum theory must either violate locality *or* CFD. Many-worlds with its multiplicity of results in different worlds violates CFD, of course, and thus can be local.

Thus many-worlds is the only local quantum theory in accord with the standard predictions of QM and, so far, with experiment.

[A] David Z Albert, *Bohm's Alternative to Quantum Mechanics* Scientific American (May 1994)
[As] Alain Aspect, J Dalibard, G Roger *Experimental test of Bell's inequalities using time-varying analyzers* Physical Review Letters Vol 49 #25 1804 (1982).
[C] John G Cramer *The transactional interpretation of quantum mechanics* Reviews of Modern Physics Vol 58 #3 647-687 (1986)
[B] John S Bell: *On the Einstein Podolsky Rosen paradox* Physics 1 #3 195-200 (1964).
[E] Albert Einstein, Boris Podolsky, Nathan Rosen: *Can quantum-mechanical description of physical reality be considered complete?* Physical Review Vol 41 777-780 (15 May 1935).
[S] Henry P Stapp *S-matrix interpretation of quantum-theory* Physical Review D Vol 3 #6 1303 (1971)

Q33 Is Everett's relative state formulation the same as many-worlds?

Yes, Everett's formulation of the relative state metatheory is the same as many-worlds, but the language has evolved a lot from Everett's original article [2] and some of his work has been extended, especially in the area of decoherence. (See "What is decoherence?") This has confused some people into thinking that Everett's "relative state metatheory" and DeWitt's "many-worlds interpretation" are different theories.

Everett [2] talked about the observer's memory sequences splitting to form a "branching tree" structure or the state of the observer being split by a measurement. (See "What is a measurement?") DeWitt introduced the term "world" for describing the split states of an observer, so that we now speak of the observer's world splitting during the measuring process. The maths is the same, but the terminology is different. (See "What is a world?")

Everett tended to speak in terms of the measuring apparatus being split by the measurement, into non-interfering states, without presenting a detailed analysis of *why* a measuring apparatus was so effective at destroying interference effects after a measurement, although the topics of orthogonality, amplification and irreversibility were covered. (See "What is a measurement?", "Why do worlds split?" and "When do worlds split?") DeWitt [4b], Gell-Mann and Hartle [10], Zurek [7a] and others have introduced the terminology of "decoherence" (See "What is decoherence?") to describe the role of amplification and irreversibility within the framework of thermodynamics.

Q34 What is a relative state?