

## Using FEniCS to solve an obstacle problem

Denote the unit square by  $\Omega$  and consider the problem

$$\begin{aligned}\max(\tfrac{1}{2}\Delta V, g - V) &= 0 \text{ on } \Omega \\ V &= 0 \text{ on } \partial\Omega,\end{aligned}$$

where  $g$  is a known function. Problems of this form are known as elliptic obstacle problems or, to probabilists, as stopping problems, since, under reasonable conditions

$$V(x) = \sup_{\text{stopping times } \tau} \mathbb{E}_{B_0=x}[g(B_\tau)],$$

where  $B$  is a two-dimensional Brownian motion. This is (informally) equivalent to minimizing the Dirichlet energy functional

$$J = \int_{\Omega} |\nabla V|^2 dx$$

over sufficiently regular  $V$  with  $V \geq g$ .

1. Can FEniCS solve this problem? This would be incredibly useful for solving optimal stopping and control problems. [1] and [2] discuss solving the elliptic obstacle problems using the FEM, but I am not experienced enough to know whether they are relevant here.
2. Are there free boundary problems which FEniCS can solve?
3. Are there other tools well suited to solving problems of this form?

[1]: [http://www.win.tue.nl/casa/meetings/seminar/previous/\\_abstract080423\\_files/ObstacleProblem.pdf](http://www.win.tue.nl/casa/meetings/seminar/previous/_abstract080423_files/ObstacleProblem.pdf)

[2]: <http://www.math.tifr.res.in/~publ/ln/tifr49.pdf> pp 95-101