Section C

1. Show that the equations x+y+z=6, x+2y+3z=14 and x+4y+7z=30 are consistent and solve them by using rank. (Mar 2006)

Answer:

The matrix equation is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 10 \end{bmatrix}$

AX=B

A= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}$; The augmented matrix is [A,B] = $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$

~ $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix}$ $R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_1$

~ $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \rightarrow R_3 - 3R_2$

∴r(A,B) = 2

Also A ~ = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$; r(A) = 2 ; r(A) = r(A,B)

But the rank is less than the number of unknowns.

The matrix equation is $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$

x+y+z = 6 -(1)

y+2z = 6

y = 8-2z -(2)

x = 6-y-z = z-2 by (2)

Taking z = k, we get

x = k-2, y = 8-2k

The solution is {k-2, 8-2k, k}

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2. Prove that cos(A+B) = cos A Cos B - Sin A Sin B by vector method.