$$
\begin{aligned}
|(z_0+h)^n - z_0^n| &= \left| \sum_{k=1}^{n} \binom{n}{k} h^k z_0^{n-k} \right| \\
&= \left| h \sum_{k=0}^{n-1} \binom{n}{k+1} h^k z_0^{n-1-k} \right| \\
&= \left| h \sum_{k=0}^{n-1} \frac{n}{k+1} \binom{n-1}{k} h^k z_0^{n-1-k} \right| \\
&\leq n|h| \sum_{k=0}^{n-1} \binom{n-1}{k} |h|^k |z_0|^{n-1-k} \\
&\leq n|h| \left(|z_0| + |h|\right)^{n-1}
\end{aligned}
$$

$$
(\alpha+\beta)\,\mathbf{u} = \alpha\mathbf{u} + \beta\mathbf{u}
$$