For $\theta^{-1}(\beta)$ we need to show that the following diagram is commutative

$$\begin{array}{ccc} f^{-1}\mathscr{G}^{\text{pre}}(U') & \xrightarrow{\theta^{-1}(\beta)(U')} & \mathscr{F}(U') \\ \Big\downarrow \text{res}^{f^{-1}\mathscr{G}^{\text{pre}}}_{U',U} & & \Big\downarrow \text{res}^{\mathscr{F}}_{U',U} \\ f^{-1}\mathscr{G}^{\text{pre}}(U) & \xrightarrow{\theta^{-1}(\beta)(U)} & \mathscr{F}(U) \end{array}$$

We have

$$\begin{aligned} \text{res}^{\mathscr{F}}_{U',U} \circ \theta^{-1}(\beta)(U') \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(U')}_{\mathscr{G}(V)} &= \text{res}^{\mathscr{F}}_{U',U} \circ \text{res}^{\mathscr{F}}_{f^{-1}(V),U'} \circ \beta(V) && \text{(by 3.3.5)} \\ &= \text{res}^{\mathscr{F}}_{f^{-1}(V),U} \circ \beta(V) \\ &= \theta^{-1}(\beta)(U) \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(U)}_{\mathscr{G}(V)} && \text{(by 3.3.5)} \\ &= \theta^{-1}(\beta)(U) \circ \text{res}^{f^{-1}\mathscr{G}^{\text{pre}}}_{U',U} \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(U')}_{\mathscr{G}(V)} && \text{(by 3.3.1)} \end{aligned}$$

By uniqueness, the above diagram is commutative. Now

$$\begin{aligned} \theta^{-1}(\theta(\alpha))(U) \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(U)}_{\mathscr{G}(V)} &= \text{res}^{\mathscr{F}}_{f^{-1}(V),U} \circ \theta(\alpha)(V) && \text{(by 3.3.5)} \\ &= \text{res}^{\mathscr{F}}_{f^{-1}(V),U} \circ \alpha(f^{-1}(V)) \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(f^{-1}(V))}_{\mathscr{G}(V)} && \text{(by 3.3.4)} \\ &= \alpha(U) \circ \text{res}^{f^{-1}\mathscr{G}^{\text{pre}}}_{f^{-1}(V),U} \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(f^{-1}(V))}_{\mathscr{G}(V)} && (\alpha \text{ is a morphism}) \\ &= \alpha(U) \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(U)}_{\mathscr{G}(V)} && \text{(by 3.3.1)} \end{aligned}$$

Therefore $\theta^{-1} \circ \theta = \text{id}_{\text{Mor}_Y(\mathscr{G}, f_*\mathscr{F})}$. Also

$$\begin{aligned} \theta(\theta^{-1}(\beta))(V) &= \theta^{-1}(\beta)(f^{-1}(V)) \circ \lambda^{f^{-1}\mathscr{G}^{\text{pre}}(f^{-1}(V))}_{\mathscr{G}(V)} \\ &= \text{res}^{\mathscr{F}}_{f^{-1}(V),f^{-1}(V)} \circ \beta(V) \\ &= \beta(V) \end{aligned}$$

and therefore $\theta \circ \theta^{-1} = \text{id}_{\text{Mor}_X(f^{-1}\mathscr{G}, \mathscr{F})}$.

Now we prove the final step. Let $\phi : \mathscr{F} \to \mathscr{F}'$ and $\psi : \mathscr{G} \to \mathscr{G}'$ be two morphism of sheaves. We have to show the following property (see 2.4.1)

$$\theta(\phi \circ \alpha \circ f^{-1}\psi) = f_*\phi \circ \theta\alpha \circ \psi$$