

In this paper I show calculations, which verify my statements. The Walsh Transform for boolean function f is defined by the following formula

$$W(f)(j) = \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i \cdot j} \quad \text{for all } j \in \{0,1\}^n,$$

where "·" denote skalar product of two vectors. Let f be the boolean function of three variables defined by truth table $(1, 0, 0, 0, 0, 1, 1, 1)$. I denote coordinates of vector i as follows $i = (i_1, i_2, i_3)$. Now I calculate Walsh Transform $W(f)$ for all $j \in \{0,1\}^n$:

$$\begin{aligned} W(f)(000) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus 0} = \\ &(-1)^1 + (-1)^0 + (-1)^0 + (-1)^0 + (-1)^0 + (-1)^1 + (-1)^1 + (-1)^1 = 0, \\ W(f)(001) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_3} = \\ &(-1)^{1+0} + (-1)^1 + (-1)^0 + (-1)^1 + (-1)^0 + (-1)^{1+1} + (-1)^{1+0} + (-1)^{1+1} = 0, \\ W(f)(010) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_2} = \\ &(-1)^{1+0} + (-1)^0 + (-1)^{0+1} + (-1)^{0+1} + (-1)^0 + (-1)^{1+0} + (-1)^{1+1} + (-1)^{1+1} = 0, \\ W(f)(011) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_2 \oplus i_3} = \\ &(-1)^1 + (-1)^1 + (-1)^1 + (-1)^2 + (-1)^0 + (-1)^2 + (-1)^2 + (-1)^3 = 0, \\ W(f)(100) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_1} = \\ &(-1)^1 + (-1)^0 + (-1)^0 + (-1)^0 + (-1)^1 + (-1)^2 + (-1)^2 + (-1)^2 = 4, \\ W(f)(101) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_1 \oplus i_3} = \\ &(-1)^1 + (-1)^1 + (-1)^0 + (-1)^1 + (-1)^1 + (-1)^3 + (-1)^2 + (-1)^3 = -4, \\ W(f)(110) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_1 \oplus i_2} = \\ &(-1)^1 + (-1)^0 + (-1)^1 + (-1)^1 + (-1)^1 + (-1)^2 + (-1)^3 + (-1)^3 = -4, \\ W(f)(111) &= \sum_{i \in \{0,1\}^n} (-1)^{f(i) \oplus i_1 \oplus i_2 \oplus i_3} = \\ &(-1)^1 + (-1)^1 + (-1)^1 + (-1)^2 + (-1)^1 + (-1)^3 + (-1)^3 + (-1)^4 = -4. \end{aligned}$$

Finally, the correct value of Walsh Transform for the function f is $(0, 0, 0, 0, 4, -4, -4, -4)$, but sage give a vector $(0, 0, 0, 0, -4, 4, 4, 4)$ as a result. All functions that I verified have in sage error result due to incorrect signs.