EN121 **Assignment 1** **Due: After the break**

PS. If I detect solution copying I will come down on you very hard. 2 same solutions = marks divided 2 ways. 3 same solutions = marks divided 3 ways, etc. If 10 of you decide to hand in the same solution [ie, one does it, the other 9 copy], each of you will get 2 marks [out of the generous 20] – if I detect the copying!

I like to make the assignments different – not just practice of what we do in lectures (that is what the tutorials are for). Something rather more difficult, and tough enough to make you think! Definitely not something you will come across in a test or exam. Something you should try to do by yourself (since the assignments will be marked generously). I have decided to expand on our class example of the motion of a damped spring, where the **top of the spring** moves sinusoidally up/down over time (with amplitude 2). I will assume our units are centimetres and seconds, and that the positive direction is up.

In this assignment will solve these **four 2nd order non-homogeneous linear DEs with constant coefficients** – each represent different spring situations:

A) $y'' + 4y' + 3y = 4\cos(t)$
B) $y'' + 4y' + 3y = 4\sin(t)$
C) $y'' + 4y' + 3y = -4\sin(t)$
D) $y'' + 4y' + 3y = -4\cos(t)$

The four solutions must each satisfy the **same** initial conditions – which is that the object is initially centred at 0 (ie, $y(0) = 0$), and is moving **up** at 5cm/sec (ie, $y'(0) = +5$).

1. Solve, fully, the first DE using hand methods – finding the general solution, then a particular solution, then matching you overall solution with the initial conditions. [8 marks]
2. Solve the other three DEs, fitting each solution to the initial conditions.
   [I am quite happy if you solve these using internet Sage (see below***)
   Note that each of the 4 DEs have the same general solution. [2 marks]
3. Draw four fairly careful sketches. Each should show an oscillation motion [respectively 1.5 cos(t), 1.5 sin(t), -1.5 sin(t), and -1.5 cos(t)] and the position of the object at time t caused by the interaction of spring and the oscillation motion.
   [You could do a cut and paste from Sage on the internet to get the graphs.] [4 marks]
4. Now sit back, look at your diagrams, and see if you can work out what is happening. Remember that the bottom of the spring is connected to the object, while the top of the spring is connected to this moving cog – and the object moves because of the stretching or contraction of the spring. Can you describe what is hapenning. [6 marks]

Note. With these coefficients the damping is (relatively) high, and (without the external motion) the movement of the object attached to the bottom of the spring dies very quickly [exponentially, in fact!] But the top of the spring ***is*** *moving up and down in a sinusoidal manner, and this will force the object to also move up and down in a type of resonance.*

*** If I wanted to solve the class problem **$y'' + 3y' + 2y = 3\cos(t)$** **$y(0) = 0$ and $y'(0) = -2$**, then draw the solution and y = 1.5cos(t), this is how I could proceed using internet sage.

1. Go to the web page sagecell.sagemath.org
2. Solve the DE with boundary conditions:

y = function('y')(x)
desolve(diff(diff (y)) + 3* diff(y) + 2*y – 3*cos(x), ics = (0, 0, -2), y, ics = (0,0,-2))

And to get the graphs:

P1 = plot(3.2*exp(-2*x) – 3.5*exp(-x) + 0.3*cos(x) + 0.9*sin(x), (0, 10))
P2 = plot(1.5*cos(x), (0,10), color='green')
P1 + P2